

Color change and pixel-matrix challenge

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Abstract

Taking advantage of a paper with one color on each face allows to produce origami models with specific patterns (for instance, John Montroll’s zebra or Satoshi Kamiya’s tiger). About geometry, some challenging models have been designed, such as a square checkerboard, as in Figure 1. To increase the difficulty, an optimality question is: what is the smallest sheet of paper to produce the same result? Using an initial square sheet of paper is then a strong requirement, due to its small perimeter-to-area ratio. A (now old) study in Dureisseix (2000) assumed that the paper edge was to be used for ensuring the many color changes, leading to an upper bound: to produce a $n \times n$ checkerboard (with n even), one needs a paper with side $n^2/2$. This assumption has been released (9 years later) in Demaine et al. (2009) and the authors exhibited a striking result: only a $n^2/4 + 4n + 4 - 5/2(n \bmod 4)$ initial side is required (for n even). This bound outperforms the previous one for $n > 16$! A general folding procedure is described, that moreover produces a seamless design (all board squares are made with a single piece of paper).

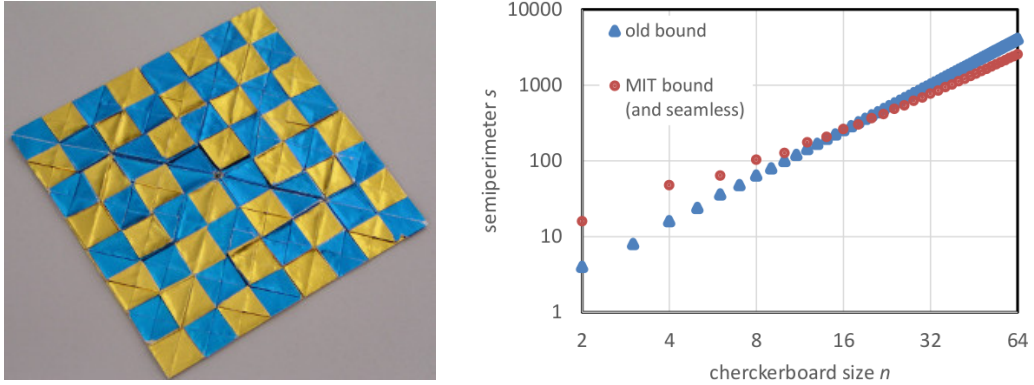


Figure 1: An optimal 8×8 chessboard and the bound comparison.

Since then, several different optimal (as far as we now) models have been effectively folded, especially for the chessboard ($n = 8$). Different solutions exist, so one can think that even more constraints can be placed on the design challenge. This communication aims to propose such a brain teaser (after again 9 years): the pixel-matrix challenge. Based on the same underlying checkerboard pattern, it now requires a color-changing mechanism (a flap) on each board square of the folded model, independently from one board square to another, to be able to shift the color of each board square, as in a matrix of pixels. A related challenge has been posted

on social networks, see [Tahir](#), but it was solved using a narrow rectangular strip of paper. Pre-scribing an initial square of paper is much more demanding. This communication is intended to provide some clues to tackle this challenge, and derives, following [Dureisseix \(2017\)](#):

- A plausible upper bound (the complexity of the folded solution) which is surprisingly the same as the static small checkerboard: a side $n^2/2$ (for n even);
- A much more complex model design issue, and a proposal for reducing its complexity: a splitting into simpler sub-problems;
- A sub-problem based on a simple flipping mechanism interpreted as a corrugation at the small scale of a 2-scale problem, Figure 2;
- A coarse scale problem interpreted as the spanning tree enumeration of a coarse graph;
- A practical trial-and-error effective folding procedure to find a stacking paper sequence (called onion layer), that appears to look like the algorithm of [Demaine and Ku \(2014\)](#);
- Up-to-date effective foldings for the static checkerboard and the pixel matrix.

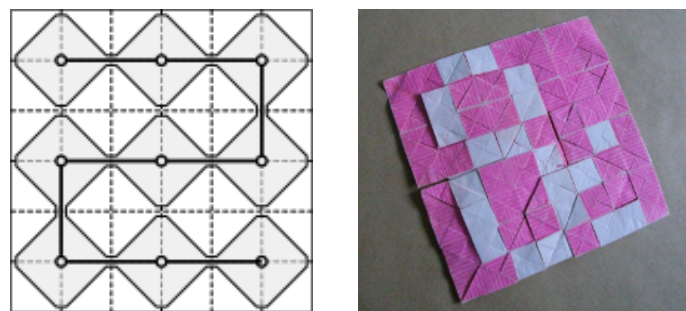


Figure 2: A 2-scale description of the 6×6 pixel-matrix (the coarse spanning tree with a thick solid line, the fine corrugation with a thin solid line) and a folded 8×8 matrix.

References

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